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ABSTRACT

This study focused on (1) the relation or transition between the measurement and partitive settings for division, and (2) the study of these two interpretations in general case where the remainder is not necessarily zero. Ninety-four second-grade pupils from four classrooms (two in each of two schools) were presented, through individual interviews, with two types of division situations. Each problem of either type was presented in an open or a partially structured setting; four problems, one of each of the possible combinations of type and setting, were presented in eight different sequences. A sequence effect was found, suggesting that, without instruction, working one type of problem interfered with rather than facilitated working with the other type. In particular, partitive problems in open settings were solved with less success if they followed other problems. Questions related to the findings are raised, as they pertain to the sequencing of content, related materials, and activities involved in conceptualization of whole-number division. (MS)

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Working Paper No. 194

A Preliminary Investigation of the Ability of Second-Grade Children to Solve Division Problems Presented in a Physical-Object Mode and in Different Partitive-Measurement Sequences

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Working Paper No. 194

A PRELIMINARY INVESTIGATION OF THE ABILITY OF SECOND-GRADE CHILDREN TO SOLVE DIVISION PROBLEMS PRESENTED IN A PHYSICAL-OBJECT MODE AND IN DIFFERENT PARTITIVE-MEASUREMENT SEQUENCES

by

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Report from the Project on
Conditions of School Learning and Instructional Strategies

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ABSTRACT

Pupil performance involving measurement and partitive problem situations presented in a physical-object mode was studied in relation to different settings (open and structured) and situation sequences. Subjects were 94 second-grade pupils from four classrooms, two in each of two schools.

A sequence effect--measurement problems followed by partitive problems, and conversely--suggested that without instruction, working one type of problem interfered with rather than facilitated working the other type. In particular, partitive problems in open settings were solved with less success if they followed other problems.

Findings from this preliminary investigation raise nontrivial questions regarding the sequencing of content and related materials and activities for effective instruction pertaining to conceptualizations of whole-number division.

I

INTRODUCTION

General Statement of the Problem

Partitive and measurement interpretations are the real-world contexts for providing meaning to whole number division. Each interpretation has been identified and studied in research studies on division. Each interpretation has been used as the means of developing and rationalizing computational algorithms for division. The general problem addressed in this study involves relations between the measurement and partitive settings. If problems from both types are presented in different sequences and different formats, are there sequence effects which facilitate and are there sequence effects which inhibit learning initial concepts in whole number division?

Measurement and Partitive Interpretations

The division exercise

$$5\overline{)21}\quad \begin{array}{l} 4 \\ \text{R } 1 \end{array}$$

can be applied in solving each of these two story problems:

> "There were 21 players and 5 members on each team. How many teams could be formed? How many players were left over?"

"Five teams were formed from 21 players. Each team had the same number of players. How many players could there be on each team? How many players were left over?"

The first story is a measurement situation. The quotient (4) answers the question "How many teams of 5 players can be formed?" The remainder (1) is the number of players left over. In the second story, the quotient in the division exercise (4) gives the number of players on each of 5 teams. This is a partitive situation. The remainder (1) gives the number of players not on one of the five teams.

In general, the division exercise

$$\begin{array}{r} q \\ b\overline{)a} \end{array} \quad R\ r$$

can be applied or have two interpretations. In the measurement interpretation, the quotient q answers the question: "How many sets of b objects?" There are a objects initially. The number of left-over objects is the remainder r. The quotient q answers a different question in the partitive interpretation: "How many per each of b "equal"[1] sets? Initially there are a objects. The remainder r gives the number of left-over objects. These two interpretations, measurement and partitive, are the situations to which one applies the mathematics of whole-number division in problem solving.

---

[1] "Equal" sets in this case technically means equivalent and disjoint sets. Also the quotient is the maximum number of objects per set.

Measurement and Partitive Division

If measurement and partitive settings are used in the development of whole number division, then these settings give different definitions or meanings to division. The term, measurement division, refers to the division process being based upon a measurement setting. Similarly, partitive division refers to tying the division process directly to a partitive setting.

Division Algorithms

The twofold nature of division carries over into the various computational procedures or algorithms for division. In this report, the term, division algorithm, will be used for any format of a specific sequence of written steps which yields the quotient and remainder for a division sentence.[2] There are essentially two types of division algorithms, the subtractive and the distributive. In developing the steps of each of these algorithms and in providing overall frameworks for the algorithms themselves, one is forced to rely on the basic measurement and partitive interpretations of

---

[2] In the literature the term, division algorithm, is also used to refer to the principle of the whole number system which states that for every (a, b) where $b \neq 0$, there exists (q, r) such that

$$a = qb + r$$

and $r < b$. However, in this report, the term, division algorithm, refers to some particular computational procedural chain.

division. The following two division exercises illustrate the essence of subtractive and distributive algorithms. Example of a subtractive algorithm:

```
23)1485          Overall question:  How many sets of 23?
 1380     60
  105            64 sets of 23, 13 left over.
   92      4
   13     64
```

Example of a distributive algorithm:

```
   64
23)1485          Overall question:  How many in each of 23 sets?
  138
  105            64 (6 tens 4 ones) in each of 23 sets, 13 left
   92                over.
   13
```

Related Research

In the past few years, several theses (Burkhart, 1967; Dilley, 1970; Kratzer, 1971) on division algorithms have been published. The distributive algorithm for division has been resurrected as a research endeavor. After more than a decade of experience with various forms of subtractive algorithms for division, both in research studies (e.g., Van Engen & Gibb, 1956) and school mathematics programs, and with recent studies on distributive algorithms (e.g., Dilley, 1970; Kratzer, 1971) at hand, it seems that one would be in a position to carefully analyze these two types of algorithms. Instructional programs of each type have been the experimental treatments of

studies. What are the research findings? In order to answer this question in an intellectually honest way, the experimental distributive and subtractive algorithm treatments need to be carefully compared in each study. Have both algorithms been carefully developed? Have the algorithm and each of the steps been rationalized in both treatments? In a sense, one would like to compare the "best" forms of the two types of algorithms when evaluating the research.

When the studies of Van Engen and Gibb (1956), Dilley (1970), and Kratzer (1971) are analyzed according to the guidelines suggested in the preceding paragraph, one does not find that two treatments of "equal quality" have been compared. The reason for these discrepancies in each case is quite understandable, reasonable, and pragmatic. Van Engen and Gibb compared a carefully developed and rationalized subtractive algorithm with a representative distributive approach as found in the school curriculum of the day. Similarly, Dilley and Kratzer in each of their studies compared their distributive algorithms with subtractive treatments in current text series. The typical subtractive algorithm in a present text contains distributive overtones and hence is much more complex than the subtractive algorithm employed by Van Engen and Gibb (1956). The subtractive algorithm employed by Van Engen and Gibb was consistently rationalized with a measurement interpretation of division, both in the overall view and in each step of the algorithm. The products in each step fit a "c sets of d" interpretation of d multiplied by c.

In both the Dilley (1970) and Kratzer (1971) studies, the overall rationalization for the distributive algorithm was a partitive interpretation of division. For example, in the exercise

$$3\overline{)72}$$

Kratzer (1971, pp. 81-83) has the quantity 72 (modeled with 7 bundles of 10 sticks and 2 "extra" sticks) partitioned evenly among three persons. The quotient gives the number for each of three sets, a partitive interpretation. The rationalization for each step of the algorithm is not fully explained. Consider this example and explanation:

$$\begin{array}{r} 2 \\ 3\overline{)72} \\ \underline{6} \\ 1 \end{array}$$

How many tens will each of the three people receive? (2) How many tens are used? (6)

Similarly, in the "ones" step

$$\begin{array}{r} 24 \\ 3\overline{)72} \\ \underline{6} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

How many ones will each receive? (4) How many ones do we use? (12)

In Dilley (1970, pp. 3-4) is found the exercise

$$\begin{array}{r} 59 \\ 23\overline{)1358} \\ \underline{115} \\ 208 \\ \underline{207} \\ 1 \end{array}$$

along with the supporting explanations:

> Put 5 bundles of 10 into each of the 23 groups.
> This uses 115 bundles of 10 and leaves 20 bundles.

One means of rationalizing the multiplication steps of a distributive algorithm using the "c sets of d" meaning for d multiplied by c is the (non-trivial) interpretation for the commutative property of multiplication. Another way to formulate this setting is to consider the "natural" transition from a measurement interpretation to a partitive interpretation, and vice versa. Van Engen and Gibb (1956, p. 128) illustrate this transition in an example for the teacher.

Measurement

There are 12 books on the table. Dick was asked to place them in piles of three to a pile. How many piles did he have?

Partition

There are 12 books on the table. Dick was asked to place them in three piles. How many would there be in each group?

(a) In each problem we have 12 books.
(b) To put the books in piles of three, we could take one group of three, another, etc.

To put the books in three groups, we could put a book in each of the groups, then another book in each of the three groups, etc. But this also results in taking 3's away from 12.

(c) In the first problem we find that we have 4 piles of books. In the second problem we find that we have 4 books in a pile.

Moser (1952) in his discussion of measurement and partitive division used repeated subtraction as the symbolism for recording both types of division at a pre-algorithmic level. Quoting from Moser's article (1952, pp. 95, 105).

> In measurement division each subtraction makes one complete group. Count the number of subtractions to find the number of groups.
>
> In partition division each subtraction places one object in each of the equal groups. Example: If 12 is to be arranged evenly into 4 groups, how many will there be in each of the equal groups? The first four subtracted from twelve will place one unit in each of the four groups required. Each subsequent subtraction will increase the size of each of the equal groups by one. The total number of subtractions will determine how many objects there will be in each of the equal groups.

Moser was aware of both a distinction between and a relation between measurement and partitive situations.

Purpose of the Study

The relation between partitive and measurement settings is relevant to instruction in division in a more fundamental way than rationalizing steps in a distributive algorithm. The partitive and measurement interpretations are the real-world bases of providing meaning for division. Hence, as a corollary, these interpretations are the means for rationalizing the algorithms. Division is applicable to both types of settings. If one division setting receives the initial emphasis in concept development and algorithmic development, then the other setting needs to be brought into the instruction as an equally important interpretation of division.

The fact that there exist two interpretations of division, partitive and measurement, has been stressed by various writers (Gunderson, 1955; Hill, 1955; Mosher, 1952; Scott, 1963; Zweng, 1963, 1964). In most of the references, the focus has been on the distinction between the two interpretations. Also, the studies relating to these two types have primarily emphasized division in the special case where the remainder is zero.

What is of interest in these two types of interpretations and what has not received much attention in the literature is

(1) the relation or transition between the measurement and partitive settings, and

(2) the study of these two interpretations in general case where the remainder is not necessarily zero.

These two items are focal points of this study.

II

METHOD

Research Design

The research design was chosen to highlight the study of relations between partitive and measurement situations. Second-grade children were selected as subjects since children at this level usually have not had systematic instruction in division with whole numbers. Since the study was conducted in early spring, these children were far along in the school year, and it was appropriate to present problem situations which are often termed pre-division activities.

The problems presented to the children were of two types, partitive or measurement. Each problem of either type was presented in an open or partially structured setting. Four problems, one of each of the possible combinations of type and setting, were presented in eight different sequences. These variables, problem type, problem setting, and four-problem sequences, are discussed in detail in the following paragraphs.

Problem Type

Two basic formats were employed:

Measurement

Start with a set of a elements.

Form sets with b elements in each set.

How many of these sets of b can be formed?

How many elements are left over?

Partitive

Start with a set of a elements.

Put the same number of elements in each of b sets.

What is the most that can be put in each of these sets?

How many elements are left over?

For these second-grade children, the elements were small blocks. Clear plastic trays were used to delineate sets of blocks. A small, red bowl was used for the left overs. Thus, the measurement and partitive formats above were phrased as follows for the children; with a and b being replaced by particular whole numbers:

Measurement

Start with these a blocks.

Put b blocks in a tray.

How many trays can be used?

Put any left overs in the red bowl.

How many left overs?

Partitive

Start with these a blocks.

Take b trays.

Put the same number of blocks in these trays.

What is the most that can be put in each tray?

Put any left overs in the red bowl.

How many left overs?

Problem Setting

Two settings were employed, an "open" setting and a "partially structured" setting. For either type problem, an open setting meant the a blocks were presented to the subject in one group. In the partially structured setting, the a blocks were presented to the child in prearranged groups.

Using the notation b for divisor, a for dividend, q for quotient, and r for remainder, facilitates describing the partially structured setting for each problem setting. The partially structured setting provided a direct tie between the measurement and partitive interpretations, and hence was one means of assessing how children relate the two types of division. In a measurement partially structured problem, the interviewer arranged the a blocks in b piles, q per pile, with r blocks scattered at the side. This pre-arrangement is a partitive interpretation available for use in a measurement problem. Each set of b can be formed by taking one block from each of the b piles; q such sets can be formed. For a partitive partially structured problem, the interviewer arranged the blocks in a measurement prearrangement. The a blocks were put in q piles, b per pile; with r blocks scattered

at the side. Each set of b blocks was one distribution for the b trays, and q such distributions were possible.

Problem Sequence

Four problems were administered to each subject. The first two were of one type, the third and fourth problems were of the other type. An open setting and a partially structured setting were used for each type of problem. Under these restrictions, there were eight possible cases for four-problem sequences. These eight cases form the basic treatments of the study and are characterized in terms of the sequence of four problems administered to a subject.

Table 1 shows the 8 four-problem sequences used in the study. The type of problem is indicated by M or P, for measurement or partitive, respectively. The problem setting is indicated by O for open or S for a partially structured setting. Thus, the notation M-O denotes a measurement problem presented in an open setting. The other letter pairs have obvious meanings.

Problems for the Study

Both a and b were specificed in each problem in accordance with these restrictions: Let q denote the quotient and r the remainder in a problem, where $b \in \{2,3,4,5,6\}$, $q \in \{2,3,4,5,6\}$, $b \neq q$, and $r \in \{0,1,2, \ldots, b-1\}$. A detailed description of how the problems were generated and assigned to the subjects is included in Appendix I.

TABLE 1

The Four-Problem Sequences Administered in the Study

Treatments	Problems			
	1st	2nd	3rd	4th
T1	M-O	M-S	P-O	P-S
T2	M-S	M-O	P-O	P-S
T3	M-O	M-S	P-S	P-O
T4	M-S	M-O	P-S	P-O
T5	P-O	P-S	M-O	M-S
T6	P-S	P-O	M-O	M-S
T7	P-O	P-S	M-S	M-O
T8	P-S	P-O	M-S	M-O

Problem Types: M - measurement, P - partitive

Problem Settings: O - open, S - partially structured

X-Y means type X and setting Y.

The problems in the study were administered individually to the subjects by an interviewer, who was also the principal investigator. An interview with each subject took from 10 to 15 minutes. The proceedings of the interview were video-taped for later scoring and coding as indicated in Appendix III.

Subjects

The subjects were the second-grade students in each of two classrooms in each of two schools. Both schools were in the same urban city which is located in a heavy industrial area. For the study, 10 was established as the minimum number of subjects per cell. Hence, 80 was the number of subjects necessary to conduct the study. These four classes had 99 children altogether.

Pilot Interviews

Prior to conducting the study as described above, a set of four pilot interviews were made with second-grade children. From these video-taped interviews, adjustments were made in the proposed procedures and a firm interview format was established. Also coding procedures for viewing the tapes and recording the data were established from this pilot study. Details for the pilot interviews are found in Appendix II.

III

RESULTS

Interviews

The interviews were conducted during a school week early in the spring. Only one of the 99 children in the four second-grade classrooms was not interviewed. This one child was absent because of illness during the entire week. Because of format errors committed by the interviewer in administering the session for three of the subjects their data were deleted from the analysis. Hence the number of subjects per treatment appeared as follows:

	T1	T2	T3	T4	T5	T6	T7	T8	Total
No. of Ss.	12	11	12	11	13	11	12	12	94

The interviews took place during the mornings at one school and during the afternoons at the other school. The interviews required using every available moment for each of the five school days.

Independent Variables

The independent variables for the main analysis of the study were:

1. Problem Type Sequence (PTS) - two measurement problems administered first or two partitive problems administered first.
2. Problem Setting Sequence (PSS) - four types determined by the open (O) and structured (S) character of the four problems: OSOS, SOOS, OSSO, SOSO.

Sex, school, and teacher variables were used as blocking variables and

reduced the error mean square. Data for these variables are in Appendix V.

Dependent Variables

The dependent variables in the main analysis were the following:

1. M-O - a score of 1 (correct) or 0 (incorrect) on the "measurement-open" problem in the sequence of 4 problems.
2. M-S - a score of 1 (correct) or 0 (incorrect) on the "measurement-partially structured" problem in the sequence of 4 problems.
3. P-O - a score of 1 (correct) or 0 (incorrect) on the "partitive-open" problem in the sequence of 4 problems.
4. P-S - a score of 1 (correct) or 0 (incorrect) on the "measurement-partially structured" problem in the sequence of 4 problems.

Variable no. 5 was an achievement measure on the four problems.

5. $\frac{1}{2}\left[(M\text{-}O) + (M\text{-}S) + (P\text{-}O) + (P\text{-}S)\right]$

Variables nos. 6 - 8 provided for a linear, quadratic, or cubic effect, respectively. A problem was given a score 1 if correct and 0 if incorrect.

6. $\frac{1}{\sqrt{20}}$ (-3 (1st prob.) - (2nd prob.) + (3rd prob.) + (4th prob.))
7. $\frac{1}{2}$ ((1st prob.) - (2nd prob.) - (3rd prob.) + (4th prob.))
8. $\frac{1}{\sqrt{20}}$ (- (1st prob.) + 3(2nd prob.) - 3(3rd prob.) + (4th prob.))

Variable no. 9 was a problem type difference and no. 10 was a problem setting difference score.

9. $\frac{1}{2}\left[(M\text{-}O) + (M\text{-}S) - (P\text{-}O) - (P\text{-}S)\right]$
10. $\frac{1}{2}\left[(M\text{-}O) + (P\text{-}O) - (M\text{-}S) - (P\text{-}S)\right]$

Variable no. 11 compared the first two problems with the last two, a linear type effect.

11. $\frac{1}{2}$ ((1st prob.) + (2nd prob.) - (3rd prob.) - (4th prob.))

Variables nos. 12 and 13 gave information on only the first two problems for the setting difference and achievement score, respectively.

12. for T1 - T4, $\frac{1}{\sqrt{2}}$ [(M-O) - (M-S)] ; for T5 - T8, $\frac{1}{\sqrt{2}}$ [(P-O) - (P-S)].

13. for T1 - T4, $\frac{1}{\sqrt{2}}$ [(M-O) + (M-S)] ; for T5 - T8, $\frac{1}{\sqrt{2}}$ [(P-O) + (P-S)].

Problem Type and Problem Setting

From Table 2 we see that cells in the basic design of the experiment themselves yielded no significant results ($p < .05$). The problem type sequence effect and the problem setting sequence effect were nonsignificant relative to success in solving the four problems. However, from Table 3 we see that problem type itself did yield significant results and that there was an interaction effect with both problem type sequence and problem setting sequence. In Table 4 we see that there was a significant result for the problem setting and also an interaction effect with the problem type sequence. From Table 5 an interaction effect is observed between problem type sequence and the interaction between problem type and problem setting.

Table 6 shows the direction of the differences. The measurement problems were solved correctly more often than were the partitive problems.

TABLE 2

ANOVA for Problem Type Sequence and Problem Setting Sequence

Source	df	MS	F	p <
Problem Type Sequence (PTS)	1	.3855	.9578	.3307
Problem Setting Sequence (PSS)	3	.3272	.8128	.4904
PTS x PSS	3	.6301	1.5656	.2040
Error Term	82	.4025		

TABLE 3

ANOVA for Problem Type (Measurement or Partitive)

Source	df	MS	F	p <
Problem Type (PT)	1	4.2553	15.7128	.0002
PT x PTS	1	1.1174	4.1260	.0455
PT x PSS	3	.6998	2.5841	.0589
PT x PTS x PSS	3	.1708	.6306	.5974
Error Term	82	.2708		

TABLE 4

ANOVA for Problem Setting (Open or Partially Structured)

Source	df	MS	F	p <
Problem Setting (PS)	1	.8617	9.4982	.0029
PS x PTS	1	.3588	3.9552	.0501
PS x PSS	3	.0694	.7651	.5169
PS x PTS x PSS	3	.0480	.5294	.6634
Error Term	82	.0907		

TABLE 5

ANOVA for Problem Type and Problem Setting Interaction

Source	df	MS	F	p <
PT x PS	1	.0106	.0992	.7536
PT x PS x PTS	1	.5922	5.5241	.0212
PT x PS x PSS	3	.0408	.3806	.7673
PT x PS x PTS x PSS	3	.1078	1.0055	.3947
Error Term	82	.1072		

Open problems were solved correctly more often than partially structured problems. The problem type sequence effect on partitive-open problem is seen in Table 7. Upon examining Tables 6 and 7, it seems that if a P-O problem were the third or fourth problem (i.e., came after first working two measurement problems) then the chances of a correct answer were not equal to that if the P-O problem were first or second. The trend favored the first or second position for a P-O problem.

In Table 8 the correlation coefficients indicate some relation exists between both measurement problems (M-O and M-S) and also between both partitive problems (P-O and P-S). However, if measurement and partitive problems are compared, there is little evidence of a strong relation.

The next three tables give information relative to the first two problems only. Hence, the subject had worked these problems of one type without possible interference from having just worked problems of the other type. From Table 9 we see that there was a significant problem setting effect. Also, from Table 10 we find that a trend is indicated relative to problem type. In Table 11 we see that subjects had greater achievement with measurement problems than partitive problems and were more successful with problems administered in an open setting than with problems administered in a partially structured setting.

Order of the Problems

The information in Table 12 suggests that there is an indication of a linear trend. The data in Table 13 concerning first and second

TABLE 6

Means of Problem Type-Sequence Scores

1st Two Problems	M-O	M-S	P-O	P-S
Measurement	.8043	.6957	.4130	.4565
Partitive	.7500	.6458	.6875	.4792

TABLE 7

ANOVA Multivariate Tests on 4 Variables M-O, M-S, P-O, P-S.

Multivariate:

Source Problem Type Sequence (PTS)

F-Ratio for multivariate test of equality of mean vectors 3.5902, df 4 and 79, $p < .0097$.

Univariate:

Variable	MS	F	$p <$
M-O	.0920	.4856	.4879
M-S	.0176	.0727	.7882
P-O	2.3204	11.4874	.0011
P-S	.0239	.1009	.7516

TABLE 8

Correlation Matrix for Problem Type-Setting Scores

	M-O	M-S	P-O	P-S
M-O	1.000			
M-S	.580	1.000		
P-O	.023	.205	1.000	
P-S	.135	.221	.518	1.000

TABLE 9

ANOVA for Problem Setting Effect on the First Two Problems Only

Source	df	MS	F	p <
Problem Setting (Pair 1, variable #12)	1	1.1968	10.4064	.0019
PS x PT	1	.1424	1.2381	.2691
Error Term	82	.1150		

TABLE 10

ANOVA for Problem Type Effect on the First Two Problems Only

Source	df	MS	F	p<
Problem Type (PT)	1	1.0305	3.1968	.0775
Problem Setting Sequence (PSS)	1	.1205	.3739	.5426
PT x PSS	1	.1654	.5132	.4758
Error Term	82	.3224		

TABLE 11

Means for the First Two Problems

Problem Setting	Problem Type		
	Measurement	Partitive	Either
Open	.8043	.6875	.7447*
Structured	.6957	.4792	.5851*
Either	.7500*	.5833*	

* Ratio of problems correct to problems worked

TABLE 12

ANOVA for Linear Order Effect

Source	df	MS	F	p <
Order of Problems (Linear, variable #6)	1	.7681	3.1795	.0783
Error Term	82	.2416		

TABLE 13

ANOVA for Linear Order Effect of the First Pair of Problems with the Second Pair of Problems

Source	df	MS	F	p <
Problem Sequence Paired (variable #11)	1	.8617	3.2349	.0758
Error Term	82	.2664		

pairs of problems support this trend. Upon examining the means displayed in Table 14 we see that, in general, a subject had more difficulty with the later problems than with the early ones in the four problem sequence. No quadratic effect (Table 15) and no cubic effect (Table 16) were detected on the order of the four problems.

TABLE 14

Means of Problems According to Order

	1st Problem	2nd Problem	3rd Problem	4th Problem
Mean	.6489	.6809	.5851	.5532

TABLE 15

ANOVA for Quadratic Order Effect

Source	df	MS	F	p <
Order of Problems (Quadratic, variable #7)	1	.0426	.4376	.5102
Error Term	82	.0972		

TABLE 16

ANOVA for Cubic Order Effect

Source	df	MS	F	p <
Order of Problems (Cubic, variable #8)	1	.1043	.7858	.3780
Error Term	82	.1327		

Methods for Solving Partitive Problems

The subjects used various approaches to solving the problems. By viewing the video-tapes the principal investigator recorded the sequence of moves taken by the subject and then classified these methods for solving the problems in the following way. (q denotes the quotient and b the divisor.)

Partitive Problems:

D1: Subject made q distributions of one block to each of the b trays.

D: Subject either made one distribution of q each to the b trays, or initially made a distribution of more than one to each of the b sets but either less than or greater than q blocks per each of the b sets.

I: Subject initially formed "unequal" sets, including the case where a large number of blocks were put in the first tray.

The subjects applied one of these methods to both problem settings. In the structured setting, the subject either used the arranged sets in some fashion or ignored the arrangement.

The data in Table 17 are recorded by grouping the first two problems and grouping the third and fourth problems. Thus, indications of differences in attacking problems before or after the other type has been encountered can be exhibited. The numbers in parentheses indicate incorrect answers to the problem. For open settings 52 out of 194 partitive problems were solved correctly. For partially structured settings, 44 out of 94 were solved correctly. Sixteen of 94 subjects used the "natural" transition method, D1, which applies a measurement method to solving a partitive problem. Six subjects used the prearranged sets in the partially structured setting in this (D1) method.

Methods for Solving Measurement Problems

The methods of attempting the measurement problems were classified as follows.

Measurement Problems:

M: Subject put b objects in a first tray, b objects in a second tray, . . ., b objects in the qth tray, and the left overs (if any) into the red bowl or the left overs were put into a $(q + 1)$st tray.

M-: Subject put b blocks in a first tray and all other blocks into the red bowl, or put b objects in more than one but less than q trays.

TABLE 17

Methods of Solving Partitive Problems

Open Problems				
Problem	Method			
	D1	D	I	Total
#1 or #2	8(0)*	22(8)	3(7)	33(15)
#3 or #4	1(0)	16(25)	2(2)	19(27)
Total	9(0)	38(33)	5(9)	52(42)

Partially Structured Problems								
Problem	Method							
	D1		D		I		Total	
#1 or #2	4(0)	0(0)**	18(19)	7(3)	1(6)	1(2)	23(25)	8(5)
		4(0)***		11(16)		0(4)		15(20)
#3 or #4	3(0)	1(0)	17(23)	7(7)	1(2)	1(2)	21(25)	9(9)
		2(0)		10(16)		0(0)		12(16)
Total	7(0)	1(0)	35(42)	14(10)	2(8)	2(4)	44(50)	17(14)
		6(0)		21(32)		0(4)		27(36)

* Notation: a (b) means a problems correct, b problems incorrect.

** Figures in this position of the table refer to problems in which the subject ignored the arrangement.

*** Figures in this position refer to problems in which the subject used the arrangement.

Quotient q, divisor b.
D1 - S made q distributions of one block to each of the b trays.
D - S made an initial distribution of 2 or more blocks to each of the b trays.
I - S initially formed "unequal" sets.

PM: Subject initially selected trays (as in a partitive problem) and then used one of the above described measurement methods.

PD: Subject initially selected trays (as a partitive problem) and then used a distributive technique in putting the blocks into these trays.

Ma: Subject put the b arranged sets of q blocks each into b trays, one arranged set per tray.

The data in Table 18 are recorded by grouping the first two problems and the third and fourth problems, as was done with the partitive problems in Table 17. Method M is the "natural" way to solve measurement problems. Forty-seven of the 94 subjects successfully used this method on the open problems. Thirty-seven used this method correctly in the partially structured settings. The data in Table 18 indicate the (partitive) distributive methods were often applied by subjects to these measurement problems if they were the third or fourth problems; i.e., came after two partitive problems.

Conservation Score

A "conservation" score was given to the subject based on the type of response given to the final interview question of "How many blocks are there altogether?" The total number of blocks, a, appeared on the cue-card board and had been explicitly stated several times during the interview. If the answer was correct and stated immediately, then the conservation score was coded 1

TABLE 18

Methods of Solving Measurement Problems

Open Problems

Problem	Method					
	M	M-	Ma	PM	PD	Total
#1 or #2	36(0)	1(9)	0(0)	0(0)	0(0)	37(9)
#3 or #4	11(2)	0(1)	0(0)	19(3)	6(6)	36(12)
Total	47(2)	1(10)	0(0)	19(3)	6(6)	73(21)

Partially Structured Problems

Problem	Method											
	M		M-		Ma		PM		PD		Total	
#1 or #2	29(2)	22(2) 7(0)	0(9)	0(6) 0(3)	1(2)	0(0) 1(2)	2(0)	1(0) 1(0)	0(1)	0(1) 0(0)	32(14)	23(9) 9(5)
#3 or #4	8(0)	6(0) 2(0)	3(3)	3(3) 0(0)	0(4)	0(0) 0(4)	14(5)	12(5) 2(0)	6(5)	4(4) 2(1)	31(17)	25(12) 6(5)
Total	37(2)	28(2) 9(0)	3(12)	3(9) 0(3)	1(6)	0(0) 1(6)	16(5)	13(5) 3(0)	6(6)	4(5) 2(1)	63(31)	48(21) 15(10)

*See footnotes of table 17 for notation used in this table, also.
Quotient q, divisor b.

M - S put b objects in 1st tray, b objects in 2nd tray,..., b objects in qth tray.
M- - S put b objects in one or more trays, but in less than q trays.
PM - S initially selected trays (as in a partitive problem) then applied one of the above methods.
PD - S initially selected trays and then applied a distributive technique.
Ma - S put the b arranged sets of q blocks each into b trays, one arranged set per tray.

(for yes), otherwise the score was 0 (for no). An obvious counting of the blocks resulted in a 0 score. Further details about this conservation score are found in Appendix III. In Table 19 the C and N refer to conservation scores of 1 and 0, respectively. The + and - notation refers to correct or incorrect solutions for the problems. For both settings of the partitive problems, there seems to be a relation between solving the problem correctly and "counting" the objects rather than relying on available quicker sources for giving the number a.

Also, a correlation coefficient was computed between X, a subject's number of 1 conservation scores, and Y, the number of problems the subject solved correctly. The correlation coefficient, $r = -0.28$, with 87 df was significant at the 0.01 level. Lower conservation scores "suggested" more correct solutions to the problems.

Size of Remainder, Divisor, and Quotient

The quotient (q) and the divisor (b) in each problem for the experiment were restricted to numbers between 2 and 6 inclusive. The remainder was selected by chance from among the numbers 0,1,2,..., b-1. Some restrictions were necessarily imposed because of the physical object format in presenting the problems to the subjects. Several checks were made about differences in even these limited exercises and corresponding success in solving the problems. In Table 20 the split between zero

Table 19

Contingency Tables for Conservation and Problem Type-Sequence

	M-O		
	+	-	Total
C	43	14	57
N	29	6	35
Total	72	20	92

$\chi^2 = .70$

For df = 1, $.50 > p > .25$

	M-S		
	+	-	Total
C	36	13	49
N	27	17	44
Total	63	30	93

$\chi^2 = 1.56$

For df = 1, $.25 > p > .10$

	P-O		
	+	-	Total
C	26	28	54
N	26	13	39
Total	52	41	93

$\chi^2 = 3.15$

For df = 1, $.10 > p > .05$

	P-S		
	+	-	Total
C	23	34	57
N	22	16	38
Total	45	50	95

$\chi^2 = 2.81$

For df = 1, $.10 > p > .05$

C - conservation score of 1

N - conservation score of 0

+ - problem correct

- - problem incorrect

TABLE 20

Contingency Table for P-0 Problems

and

Zero vs. Nonzero Remainder

Remainder	Problem		
	Correct	Incorrect	Total
Zero	12	17	29
Nonzero	40	25	65
Total	52	42	94

$\chi^2 = 3.293$

For df = 1, .10> p> .05

remainder and nonzero remainder seems to have a relation with correct and incorrect P-0 problems. Problems having a nonzero remainder were solved more successfully than those P-0 problems having a zero remainder.

In Tables 21 and 22, we see that P-S problems with the divisor or quotient, and especially the divisor, equal to 2 were easier than the other P-S problems.

TABLE 21

Contingency Table for P-S Problems

and

Either the Divisor or Quotient Equal 2

b, q	Problem		
	Correct	Incorrect	Total
b = 2 or q = 2	29	18	47
b ≠ 2 and q ≠ 2	15	32	47
Total	44	50	94

$\chi^2 = 8.379$

For df = 1, p ≤ .005

b - divisor

q - quotient

TABLE 22

Contingency Table for P-S Problems Having Quotient

or

Divisor Equal to 2

b, q	Problem		
	Correct	Incorrect	Total
$b = 2, q \neq 2$	19	5	24
$q = 2, b \neq 2$	10	13	23
Total	29	18	47

$\chi^2 = 7.72$

For df = 1, $.01 > p > .005$

b - divisor

q - quotient

IV

INTERPRETATIONS

Preliminary Nature of the Study

In a preliminary investigation, such as was this study, one cannot make definitive assertions. However, the results of the study indicate a few trends which should be noted, and which could be subjected to further refined research. The comments and assertions given in this interpretations section should be read within this formative framework.

The subjects or pupils in this study were given division (or pre-division) problems prior to receiving systematic instruction in division. Hence, in interpreting the results of this study one can consider those aspects which show a base or point of departure (readiness behaviors) for instruction in division, and those aspects which are danger signals alerting curriculum writers and teachers to those areas which require a careful instructional program in division.

Problem Type and Problem Setting

The problem type effect within the basic design of the experiment reinforces the results from other studies that partitive and measurement problems are "different" as viewed by the child. The other aspects of the basic design provided several ways of examining relations between these two types of problems.

The problem setting provided the pupil with a means of applying the solution for one problem type in answering a problem of the other type.

A pupil who was aware of this relation could easily solve a structured problem. The results of this study show that without instruction these applications from one type to another interfere, not assist, in solving these problems.

The sequence effect of following measurement problems by partitive problems, and conversely, indicated that, again, without instruction working both types of problems causes interference, not facility in working the second type. In particular, the partitive-open problems, were solved with less success if they followed other problems.

Problem Solving Methods

From the analysis of the methods used in attacking the problems it seemed that the procedures employed for 3rd and 4th problems were adversely affected by the preceding problems of the other type. However, in examining the strategies for the measurement or partitive problems appearing as the first pair in the sequence, a few trends can be noted. The measurement problems were attacked in a common and natural way. Sets of b each were formed. What to do with left-overs seemed to cause some problems. Also, if there were no left-overs the presence of the red bowl seems to force some pupils into placing some blocks in the red bowl, although the interviewer stated "Put any left-overs in the red bowl". The results of the study seem to support that measurement problems can be solved in an easy way: Make as many sets of b each as possible.

The methods of attacking partitive problems provided interesting data for inference and speculation. The standard way of solving partitive problems, and thereby apply a measurement solution to solving a partitive problem, is to apply the method categorized in this study as D1. In

this method a subject starts with a objects. A set of b objects is used to make a distribution of one object to each of the b sets. This process is repeated until no more sets of b objects can be formed. Hence, the number of sets of b (measurement method) gives the number of objects for each of the b sets (partitive application). This method was employed by some pupils, but it was not the most common method of attack, and was not facilitated by the explicit structured setting. Methods in which the initial distribution was more than one for each of the b sets was the common approach, method D. If D1 is the approach to be used in a mathematics program the D approach can be modified, but instruction is necessary to develop this application of using a measurement setting to solve a partitive problem.

Secondary Results

A very minor consideration in the study was the matter of the conservation score. In coding this score the investigator would only mark "yes" if there was a quick response. Any delay in response resulted in a "no" mark. Hence there may have been subjects who could or even did "conserve" but were scored otherwise. The fact that this conservation score correlated negatively with problem success, especially with partitive problems, raises a minor point which might be of interest in further study. Perhaps methodical counters are more successful in solving partitive problems presented in a physical object format, than the subject who quickly relies on previous information and is prone to take "short cuts."

Another minor consideration involved the zero and non-zero remainder dichotomy for the problems. It seemed that with the general format of

requiring a remainder answer ("Put any left-overs in the red bowl.") a division problem with a non-zero remainder seemed easier to the pupils than one with a zero remainder. Pupils were somewhat reluctant to leave the red bowl empty.

A third minor point involved the size of the quotient or divisor. In the partitive structured problems it seemed that the exercises in which the divisor was two were quite easy compared to other exercises. In a P-S problem with divisor 2 the pupil started with a prearranged sets of 2 blocks. Pupils used these blocks to "fill" two trays. Some "dealt" one each from each pile to the two trays. Others distributed by pairs, splitting the last arranged set if necessary. "Easy" problems of this type might have an instructional value in introductory treatments of division.

Summary and Extrapolation

This study was conducted in a pre-division setting. The subjects had not received systematic instruction in the symbolism for division nor had they received systematic instruction in using physical objects in answering "How many sets?" (measurement) and "How many in each set?" (partitive) questions. Classroom instruction for pre-division topics should and often does include both measurement and partitive activities. The findings in this study suggest that, without instruction, children often employ the "natural method" (labeled M in this study) for solving measurement problems. Also, again without instruction, children often use a distribution method (D1 or D) in solving a partitive problem. But a distribution method implies using a measurement approach to solving a

partitive problem. Hence, with instruction what directions seem to be reasonable for a classroom program in developing pre-division and division topics? The following questions, extrapolated from this study, are posed for consideration in developing curriculum materials for division with whole numbers and as sources of conjectures for researchers in mathematics education:

Should the first pre-division activities be measurement problems and should the method M be taught as the way to solve these problems?

Should partitive pre-division activities be postponed until measurement problems can be readily solved?

Should the method D1 be taught as the means for solving a partitive problem?

Should the measurement approach in method D1 be made explicit in the instruction?

Should the measurement setting be the real world basis for giving meaning to division with whole numbers?

How should division with whole numbers be related to partitive situations?

How should a division algorithm be rationalized so that the overall setting, steps in the algorithm, and the relations between these two components are meaningful?

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Appendix I

Generating and Assigning Problems to Treatments

A list of 416 problems were generated for the study by using a table of random digits. Entry into the table was made in accordance with the instructions given in the book. The first digit between (inclusive) 2 and 6 was the divisor b in the first problem in the list. The next digit between 2 and 6 and not equal to b was the quotient q. The next digit greater than or equal to 0 but less than b was the remainder, r. Continuing with the sequence of digits, this process was repeated for finding b, q, and r for the next problem. Proceeding in similar fashion a list of 416 problems was constructed. Since each subject was to receive 4 problems, the first four problems of the list were grouped and labeled no. 1, the next four became the no. 2 group, etc., until 104 groups of 4 problems had been designated.

The table of random digits was also used to assign these 104 sets of 4 problems to the 8 treatment groups. The table of random digits was used to generate a permutation of the numbers 1 through 80, and 3 other permutations of 1 through 8. Let p_1, p_2, p_3, p_4 be these permutations, respectively, The assignment to the eight cells can be described as follows:

Appendix I (cont.)

x is the group no. for a set of 4 problems

m_8 maps a number modulo 8

Group No.	Treatment No.
For $1 \leq x \leq 80$	$m_8 [p_1(x)]$
For $81 \leq x \leq 88$	$m_8 [p_2(x-80)]$
For $89 \leq x \leq 96$	$m_8 [p_3(x-88)]$
For $97 \leq x \leq 104$	$m_8 [p_4(x-96)]$

The problems were administered to the subjects according to the group number for the problems. The first subject interviewed was given problem no. 1, etc.

Appendix II

Trial Experiment

Prior to conducting the study a trial run of the interview procedures was made with four second grade children from a school not used in the study. From this trial run it was established that a ten to fifteen minute period was sufficient for one interview, the procedures of the interview needed to be revised slightly, and a coding format for evaluating the video tapes was devised. The principal investigator conducted the interviews in a room furnished with a remote control TV camera. All three of the investigators viewed the video tape of the interviews and developed the revised interview format and a set of coding procedures for the study.

Originally the subject was to be given two warm-up exercises before the 4 problems of the study were administered. From this pilot work, it was found that one such exercise sufficed. Five blocks were put in each of 6 trays and 4 blocks were put into the red bowl. The child was asked:

> How many trays are there?
> Are there the same number of blocks in each tray?
> How many blocks are in each tray?
> How many blocks are in the red bowl?

This series of questions established the terminology for the experiment and also established that the child could count, at least to six. If a child could not answer these questions correctly he or she was to be deleted from the study. All of the children in the trial run and in

Appendix II (cont.)

the main study could answer these questions.

The trays for the experiment were clear plastic "sandwich savers". The clear bottoms for the trays made it somewhat difficult to see what blocks were in which trays in the video tape. Placing white paper squares on the bottoms of the trays solved this problem.

One major outcome of the trial run was the awareness that the problems needed to be presented as division problems, not a sequence of procedures in a laboratory or activity setting. Hence the revised format included having the interviewer first read the entire problem, then repeat the various parts of the problem verbatim, as needed. The statements for the problem were on a cue-card board which the pupil could read. The interviewer read from the cue-card, to insure uniformity in presenting problems to different children. After the trial run the instructions on the cue-card boards (one for partitive, and one for measurement problems) were revised to read as listed in the Method Section of this report. Another change was implemented which also aided in highlighting the problem to the subject. After the child had finished solving the problem with the blocks and trays it was decided to have the interviewer ask summary questions:

"Starting with these a blocks and putting b blocks in each tray (measurement problem), how many trays can be used? How many left-overs are there? How many blocks are there all together?"

(The interviewer was to indicate with a sweeping motion that this last question referred to the trays and red

bowl). Analogous questions were asked following a partitive problem, also. The method of answering the last question became the basis for the conservation score.

Since one of the children solved these problems by looking at the objects, mentally solving the problem, and then placing the objects in the trays according to his solution, it was decided to assess the subjects along two categories:

(1) Mediation - Did the child use the objects as a mediating device in solving the problem?

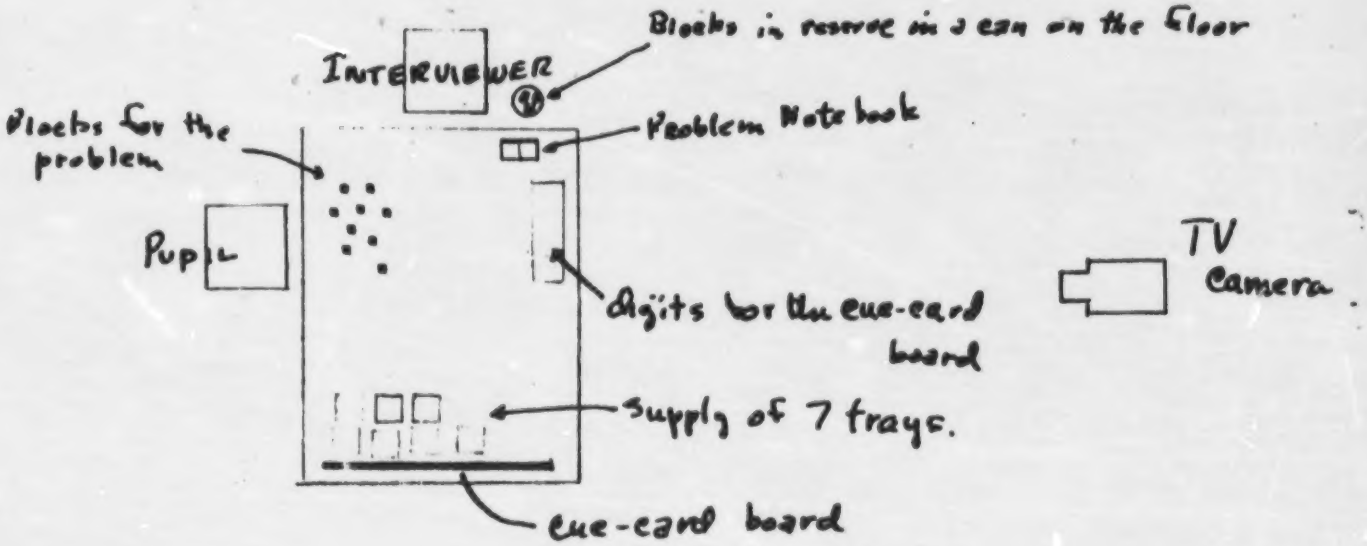
(2) Application - Did the child solve the problem mentally and then apply this solution and move the objects accordingly?

Classifying subjects in the study along this dimension was difficult and this aspect of the study was eventually abandoned.

Appendix III

Subject Interview and Coding the Results

The subjects were interviewed individually in a small room. In one school the room was adjacent to the principal's office. The other room was located in a more acceptable out-of-the-way area of the second school. The subject and interviewer worked at a low table with the TV camera located on a tripod 6 to 10 feet away. The interviewer was also the TV cameraman. The children obviously noticed the camera, but did not seem to be adversely affected by its presence. The interviewer mentioned that the TV camera allowed the work to be done without the child (and interviewer) doing any writing. The diagram shows the interview setting.



Appendix III

The interviewer used a small notebook. On each page was a set of four problems for the subject being interviewed. Problems were stated concisely and in a form which would not be readily understood by the subject, should he be able to see the sheet. For example

P-O (2,6,1) - 13

denotes a partitive open problem with $\underline{b} = 2$, $\underline{q} = 6$, $\underline{r} = 1$ and $\underline{a} = 13$.

During the preliminary part of the interview the child was asked to give his or her name which the interviewer wrote on the notebook sheet. Making sure that the name was spelled correctly, etc., gave the interviewer a chance to "break the ice". Then the subject was asked to work the warm-up exercise which is described in Appendix II. Then the first problem was "set up". The blocks were properly arranged in front of the pupil and the measurement or partitive cue-card board was completed by inserting the proper digits for the problem. The digits were on cardboard backing having scotch-tape tabs for easy application and removal from the board. The TV camera was switched on by the interviewer and the main part of the session had begun.

The problem appearing on the cue-card board was read in its entirety to the subject and then repeated as necessary to help the student in understanding the problem. The interviewer adhered to reading the problem verbatim each time. After the subject solved or had completed "his solution" by indicating that the blocks were where he wished to place them the interviewer recapitulated stating the problem

(see Appendix II) and ended this problem with the question, "How many blocks are there all together?" Then the blocks and trays were arranged for the next problem, and the appropriate digits were put on the cue-card board. The procedure described above was repeated for this problem. Then the entire procedure for a problem was repeated for each of the 3rd and 4th problems. And finally the camera was switched off, the interview having been completed. The only written part for the entire interview was the recording of the subject's name on the notebook problem set sheet.

After the interview was completed the pupil went back to the classroom and informed the teacher that the "man" was ready for the next person. All of the pupils in one classroom were interviewed at a school before progressing to the pupils in the other classroom. Also, the teacher determined the order in which the pupils were interviewed. Because of the way in which the subjects were assigned to treatments (see Appendix I), it was insignificant as to the order in which the pupils were interviewed.

The principal investigator coded the results by viewing the video-tapes and recording the results on a form sheet. The information recorded for a problem is illustrated by the following example:

M - 0 (2, 6, 0) - 12

Problem: Sequence of procedures:

X correct _ incorrect M

Appendix III

Conservation:

X yes _ no

Role of objects:

X mediation

_ application

A problem was scored correct if the child solved the problem either before the recapitulation questions of the interview or if last minute correction changes were made because of these questions. The "conservation" check and "role of the objects" assessment were explained in Appendix II. The latter was finally aborted. The symbol under the "Sequence of procedures" item is a code for a method category. These methods were identified in the Results section of this report. In the example, M refers to the pupil putting 2 blocks in a tray, one tray at a time, until six trays have been used. In coding the method used by the pupil, the investigator wrote a detailed description of the steps used by the pupil if the method could not be readily classified according to a previously used code abbreviation.

Appendix IV Raw Data

Sub. No.	Tr. No.	Sch. No.	Teac.	Sex	M-O Problem Prob.	b	q	r	a	score	cons.	M-S Problem Prob.	b	q	r	a	score	cons.
1	3	1	E	F	1st	2	4	1	9	0	1	2nd	5	2	0	10	0	1
2	3	1	E	F	1st	5	4	2	22	1	1	2nd	4	2	3	11	1	1
3	4	1	E	F	2nd	3	5	2	17	1	1	1st	6	5	5	35	0	0
4	Blocks not prearranged in P-S problem.																	
5	1	1	E	F	1st	2	6	0	12	1	1	2nd	5	4	3	23	0	0
6	6	1	E	F	3rd	5	4	2	22	1	1	4th	4	2	2	10	1	1
7	4	1	E	M	2nd	4	2	1	9	0	0	1st	5	4	1	21	0	0
8	8	1	E	M	4th	5	4	4	24	0	0	3rd	3	6	0	18	0	0
9	6	1	E	M	3rd	5	3	3	18	1	1	4th	4	2	0	8	1	1
10	7	2	L	M	4th	5	3	0	15	1	1	3rd	3	4	0	12	0	1
11	1	2	L	M	1st	2	3	1	7	0	1	2nd	4	3	0	12	0	1
12	5	2	L	M	3rd	5	4	4	24	1	0	4th	3	5	1	16	0	0
13	4	2	L	M	2nd	5	3	3	18	1	0	1st	3	6	2	20	1	0
14	Blocks arranged incorrectly on M-S problem.																	
15	2	2	L	M	2nd	4	2	1	9	1	1	1st	4	6	1	25	1	1
16	6	2	L	M	3rd	4	5	1	21	1	1	4th	4	3	3	15	1	1
17	2	2	L	M	2nd	6	5	4	34	1	1	1st	5	2	1	11	1	1
18	8	1	E	F	4th	5	4	4	24	1	1	3rd	4	3	2	14	1	1
19	4	1	E	F	2nd	3	4	2	14	1	0	1st	4	2	2	10	1	0
20	1	1	E	M	1st	3	4	2	14	0	1	2nd	2	4	1	9	0	1
21	7	1	E	M	4th	5	6	0	30	1	1	3rd	5	2	3	13	1	1
22	6	1	E	M	3rd	2	4	1	9	1	1	4th	5	6	3	33	1	1
23	3	1	E	M	2nd	2	4	1	9	1	0	1st	6	5	2	32	1	0
24	Blocks arranged incorrectly in P-S problem.																	
25	7	1	E	M	4th	6	3	4	22	0	1	3rd	5	3	1	16	1	0

Appendix IV (continued)

Sub.	P-O Problem							P-S Problem							Total
No.	Prob.	b	q	r	a	score	cons.	Prob.	b	q	r	a	score	cons.	score
1	4th	4	2	2	10	0	1	3rd	6	5	3	33	0	1	0
2	4th	3	2	0	6	0	1	3rd	2	6	0	12	0	1	2
3	4th	6	3	3	21	0	1	3rd	6	2	2	14	0	1	1
4															
5	3rd	6	5	1	31	0	1	4th	4	3	0	12	0	1	1
6	2nd	4	5	2	22	0	1	1st	2	4	0	8	1	1	3
7	4th	2	4	0	8	0	0	3rd	3	4	2	14	0	0	0
8	2nd	5	3	3	18	0	0	1st	2	3	1	7	0	0	0
9	2nd	3	5	2	17	1	1	1st	4	5	2	22	1	1	4
10	1st	5	2	2	12	1	0	2nd	3	6	0	18	1	1	3
11	3rd	4	2	3	11	1		4th	4	2	3	11	0	1	1
12	1st	6	5	2	32	1	0	2nd	4	6	2	26	0	0	2
13	4th	3	2	1	7	1	0	3rd	2	5	0	10	1	0	4
14															
15	3rd	5	4	3	23	1	1	4th	2	6	0	12	1	1	4
16	2nd	6	2	2	14	1	1	1st	4	6	3	27	0	0	3
17	3rd	5	4	0	20	1	1	4th	5	6	0	30	0	1	3
18	2nd	5	4	3	23	1	1	1st	6	3	1	19	0	1	3
19	4th	3	6	2	20	1	0	3rd	5	6	2	32	1	0	4
20	3rd	3	4	2	14	0	1	4th	6	4	0	24	0	1	0
21	1st	6	5	3	33	1	0	2nd	5	6	3	33	1	0	4
22	2nd	2	5	0	10	1	0	1st	3	2	2	8	0	0	3
23	3rd	6	4	0	24	0	0	4th	3	2	0	6	0	0	2
24															
25	1st	6	4	0	24	1	1	2nd	2	3	0	6	1	1	3

A blank indicates that this information was not available. (The interviewer failed to ask the pertinent question.)

Sub. No.	Tr. No.	Sch. No.	Teac.	Sex	M-O Problem							M-S Problem						
					Prob.	b	q	r	a	score	cons.	Prob.	b	q	r	a	score	cons.
26	2	1	E	M	2nd	2	5	1	11	1	0	1st	4	5	0	20	1	0
27	8	2	L	M	4th	6	4	4	28	0	1	3rd	5	4	0	20	1	1
28	3	2	L	M	1st	2	3	0	6	1	0	2nd	2	4	0	8	1	0
29	3	2	L	M	1st	2	3	0	6	1	1	2nd	4	5	3	23	1	1
30	1	2	L	M	1st	6	4	1	25	1	1	2nd	6	3	0	18	1	1
31	8	2	L	F	4th	5	6	2	32	1	1	3rd	4	6	2	26	0	1
32	2	2	L	F	2nd	5	2	4	14	1	1	1st	5	6	1	31	0	1
33	8	2	L	F	4th	5	2	3	13	1	1	3rd	6	2	3	15	1	1
34	7	2	L	F	4th	4	2	2	10	1	0	3rd	5	2	3	13	1	0
35	5	2	L	F	3rd	5	6	2	32	1	1	4th	2	6	1	13	1	1
36	5	2	L	F	3rd	4	2	3	11	1	0	4th	3	4	0	12	0	0
37	4	1	E	M	2nd	3	5	1	16	1	0	1st	4	5	2	22	1	0
38	4	1	E	M	2nd	3	5	1	16	1	0	1st	4	5	2	22	1	0
39	6	1	E	M	3rd	2	6	0	12	1	1	4th	6	4	2	26	1	0
40	7	1	E	M	4th	5	3	4	19	1	0	3rd	5	4	0	20	0	0
41	8	1	E	M	4th	3	6	1	19	1	0	3rd	2	3	0	6	1	1
42	6	1	W	M	3rd	6	2	2	14	0	0	4th	4	6	3	27	0	1
43	4	1	W	M	2nd	4	3	0	12	1	1	1st	5	2	0	10	1	1
44	3	1	W	M	1st	6	2	4	16	1	1	2nd	4	3	0	12	1	1
45	7	1	W	M	4th	6	4	2	26	1	1	3rd	5	3	0	15	1	1
46	5	2	L	F	3rd	4	6	2	26	0	1	4th	5	6	0	30	0	1
47	5	2	L	F	3rd	5	3	1	16	1	0	4th	3	5	2	17	1	0
48	2	2	L	F	2nd	5	2	2	12	0	1	1st	4	2	3	11	0	1
49	8	2	L	F	4th	6	3	3	21	1	0	3rd	2	6	1	13	1	1
50	1	2	L	M	1st	4	3	2	14	1	1	2nd	3	2	0	6	1	1

Sub.	P-O Problem							P-S Problem							Total
No.	Prob.	b	q	r	a	score	cons.	Prob.	b	q	r	a	score	cons.	score
26	3rd	2	4	1	9	1	0	4th	5	2	2	12	1	0	4
27	2nd	3	5	0	15	1	1	1st	5	4	4	24	0	1	2
28	4th	2	6	0	12	0	0	3rd	6	3	0	18	0	0	2
29	4th	3	4	1	13	0	1	3rd	6	3	0	18	0	1	2
30	3rd	4	5	1	21	0	1	4th	6	2	4	16	0	1	2
31	2nd	2	3	0	6	0	0	1st	5	2	4	14	0	1	1
32	3rd	6	2	5	17	1	1	4th	2	6	0	12	1	1	3
33	2nd	6	3	1	19	0	1	1st	3	2	0	6	0	1	2
34	1st	4	3	3	15	1	0	2nd	3	2	0	6	1	1	4
35	1st	4	3	1	13	1	0	2nd	5	4	0	20	1	1	4
36	1st	4	6	0	24	0	0	2nd	4	6	2	26	0	0	1
37	4th	6	5	1	31	0	1	3rd	2	6	1	13	1	1	3
38	4th	5	3	1	16	0	1	3rd	3	2	1	7	0	1	2
39	2nd	5	2	0	10	1	1	1st	4	5	0	20	1	0	4
40	1st	5	6	0	30	0	0	2nd	6	4	0	24	0	1	1
41	2nd	6	3	4	22	0	1	1st	6	2	2	14	0	1	2
42	2nd	6	5	4	34	0	0	1st	2	5	0	10	0	0	0
43	4th	3	6	0	18	0	1	3rd	5	3	4	19	0	1	2
44	4th	3	2	1	7	0	1	3rd	5	2	4	14	0	1	2
45	1st	3	4	1	13	1	0	2nd	4	5	0	20	0	1	3
46	1st	2	4	0	8	1	1	2nd	4	6	3	27	0	1	1
47	1st	2	4	1	9	0	1	2nd	6	2	2	14	1	0	3
48	3rd	2	4	0	8	1	1	4th	2	5	0	10	1	1	2
49	2nd	5	4	3	23	1	1	1st	5	3	3	18	1	1	4
50	3rd	3	6	2	20	0	1	4th	3	5	0	15	0	1	2

Sub. No.	Tr. No.	Sch.	Teac.	Sex	M-O Problem							M-S Problem						
					Prob.	b	q	r	a	score	cons.	Prob.	b	q	r	a	score	cons.
51	5	2	L	M	3rd	5	2	2	12	1		4th	2	3	1	7	0	0
52	6	2	H	M	3rd	2	5	0	10	0	1	4th	4	5	4	12	0	1
53	2	2	H	M	2nd	5	2	2	12	1	1	1st	6	3	5	23	0	0
54*	2	2	H	M	2nd	2	5	1	10	1	1	1st	6	2	1	13	1	1
55	3	2	H	M	1st	5	2	2	12	0	1	2nd	6	5	3	33	0	0
56	7	2	H	M	4th	6	5	4	34	1	0	3rd	5	6	3	33	1	0
57	8	1	W	F	4th	6	3	1	19	0	0	3rd	2	3	1	7	0	1
58	5	1	W	M	3rd	6	2	4	16	1	0	4th	5	6	0	30	1	0
59	1	1	W	F	1st	4	2	2	10	1	0	2nd	6	2	1	13	1	0
60	1	1	W	M	1st	5	6	1	31	1	1	2nd	4	2	2	10	1	1
61	4	1	W	M	2nd	6	3	3	21	1	1	1st	6	3	4	22	1	1
62	3	1	W	M	1st	2	4	1	9	1	1	2nd	3	2	2	8	1	0
63	6	1	W	F	3rd	3	2	2	8	1	1	4th	5	4	2	22	1	1
64	8	1	W	M	4th	2	5	1	11	1	1	3rd	3	4	2	14	1	1
65	7	1	W	F	4th	4	2	1	9	1	1	3rd	4	3	2	14	0	1
66	7	1	W	M	4th	6	5	4	34	0	1	3rd	4	6	3	27	1	0
67	1	2	H	M	1st	4	6	1	25	1	0	2nd	3	5	1	16	1	0
68	5	2	H	M	3rd	2	3	1	7	1	1	4th	4	3	1	13	1	1
69	2	2	H	M	2nd	4	6	1	25	1	0	1st	4	5	3	23	1	0
70	3	2	H	M	1st	2	5	1	11	1		2nd	5	3	1	19	1	1
71	2	2	H	M	2nd	6	2	4	16	1	1	1st	2	6	1	13	1	1
72	3	2	H	M	1st	3	4	2	14	1	0	2nd	3	2	2	8	1	0
73	7	2	H	M	4th	3	5	1	16	1	1	3rd	4	5	2	22	1	0
74	4	2	H	M	2nd	3	4	0	12	0	1	1st	4	2	3	11	0	0
75	3	2	H	M	1st	6	2	1	13	1	0	2nd	2	4	1	9	1	0

* Subject No. 54's data were deleted from ANOVA since this subject should have been T2, but was administered a T4 sequence of problems.

Sub.	P-O Problem							P-S Problem							Total
No.	Prob.	b	q	r	a	score	cons.	Prob.	b	q	r	a	score	cons.	score
51	1st	2	6	1	13	1	0	2nd	2	5	0	10	1	0	3
52	2nd	5	6	0	30	1	0	1st	2	3	1	7	1	0	2
53	3rd	2	6	0	12	0	1	4th	6	2	4	16	0	1	1
54	4th	6	4	5	29	1	0	3rd	2	3	0	6	1	1	4
55	4th	3	2	0	6	0	0	3rd	2	5	0	10	0	0	0
56	1st	2	3	1	7	1	0	2nd	6	5	3	33	0	0	3
57	2nd	3	5	2	17	0	0	1st	2	6	1	13	1	0	1
58	1st	3	2	2	8	1	0	2nd	6	4	2	26	1	0	4
59	3rd	2	4	1	9	1	1	4th	2	3	1	7	1	1	4
60	3rd	2	6	1	13	0	1	4th	5	6	0	30	0	1	2
61	4th	6	3	2	20	0	1	3rd	4	6	1	25	1	1	3
62	4th	4	3	1	13	1	1	3rd	2	5	1	11	1	0	4
63	2nd	3	2	0	6	1	1	1st	5	6	2	32	0	0	3
64	2nd	5	3	2	17	1	1	1st	5	2	0	10	1	1	4
65	1st	5	6	0	30	0	0	2nd	6	5	4	34	0	1	1
66	1st	6	5	5	35	0	1	2nd	6	4	0	24	0	0	1
67	3rd	6	3	5	23	1	0	4th	3	2	2	8	1	1	4
68	1st	3	4	1	13	1	0	2nd	4	6	0	24	1	1	4
69	3rd	4	2	1	9	1	0	4th	5	3	3	18	1	0	4
70	4th	5	6	1	31	0	1	3rd	4	3	0	12	0	1	2
71	3rd	2	3	0	6	1	1	4th	6	2	1	13	1	1	4
72	4th	2	5	0	10	0	1	3rd	3	5	0	15	0	1	2
73	1st	2	4	1	9	1	0	2nd	4	6	3	27	0	1	3
74	4th	2	5	1	11	1	1	3rd	2	4	1	9	1	1	2
75	4th	4	3	2	14	1	0	3rd	2	6	0	12	1	0	4

Sub. No.	Tr. No.	Sch. No.	Teac.	Sex	M-O Problem							M-S Problem						
					Prob.	b	q	r	a	score	cons.	Prob.	b	q	r	a	score	cons.
76	1	2	H	M	1st	4	3	1	13	1	1	2nd	3	5	0	15	1	1
77	5	1	W	F	3rd	4	2	2	10	1	1	4th	4	5	2	22	1	0
78	6	1	W	M	3rd	6	3	4	22	0	1	4th	2	5	1	11	0	0
79	5	1	W	F	3rd	5	2	4	14	1	0	4th	5	2	3	13	1	0
80	8	1	W	M	4th	6	4	2	26	0	1	3rd	2	6	1	13	0	1
81	4	1	W	M	2nd	4	3	1	13	1	0	1st	6	4	1	25	1	0
82	8	1	W	M	4th	5	4	2	22	1	0	3rd	4	3	1	13	0	0
83	6	1	W	F	3rd	2	3	0	6	0	0	4th	2	4	1	9	0	0
84	7	1	W	M	4th	2	5	1	11	1	1	3rd	6	4	4	28	1	1
85	1	1	W	M	1st	4	3	1	13	1	0	2nd	5	6	4	34	1	0
86	2	1	W	F	2nd	6	5	0	30	1	1	1st	3	4	2	14	0	0
87	5	1	W	F	3rd	5	2	1	11	1	1	4th	2	3	0	6	1	1
88	3	1	W	F	1st	3	5	1	16	1	1	2nd	3	2	1	7	1	1
89	2	2	H	F	2nd	6	5	4	34	1	0	1st	4	5	1	21	0	0
90	4	2	H	F	2nd	4	3	3	15	1	0	1st	6	5	5	35	1	
91	6	2	H	F	3rd	4	6	0	24	1	1	4th	2	4	0	8	1	1
92	5	2	H	F	3rd	2	4	0	8	1	0	4th	3	2	2	8	1	1
93	8	2	H	F	4th	5	4	4	24	1	1	3rd	4	2	0	8	1	1
94	7	2	H	F	4th	5	4	0	20	1	0	3rd	2	6	1	13	1	0
95	3	2	H	F	1st	4	3	2	14	0		2nd	2	3	0	6	0	1
96	1	2	H	F	1st	3	5	0	15	0	1	2nd	3	2	2	8	1	1
97	1	2	H	F	1st	6	5	2	32	1	1	2nd	4	2	1	9	1	0
98	5	2	H	F	3rd	6	2	0	12	0	0	4th	2	6	1	13	0	0

Sub.	P-O Problem							P-S Problem							Total
No.	Prob.	b	q	r	a	score	cons.	Prob.	b	q	r	a	score	cons.	score
76	3rd	6	2	2	14	1	1	4th	2	3	1	7	1	0	4
77	1st	6	5	1	31	0	0	2nd	4	6	0	24	0	1	2
78	2nd	2	4	1	9	1	1	1st	4	2	3	11	0	0	1
79	1st	3	4	2	14	1	0	2nd	2	4	1	9	1	0	4
80	2nd	5	6	1	31	0	1	1st	5	6	4	34	0	1	0
81	4th	5	6	0	30	0	1	3rd	6	3	4	22	0	0	2
82	2nd	6	3	2	20	1	0	1st	2	4	0	8	1	1	3
83	2nd	5	3	4	19	1	0	1st	3	2	2	8	1	0	2
84	1st	6	5	0	30	1	0	2nd	6	4	0	24	0	1	3
85	3rd	2	3	0	6	0		4th	4	5	1	21	0	0	2
86	3rd	4	6	0	24	0	0	4th	4	6	2	26	1	0	2
87	1st	4	6	0	24	1	1	2nd	4	5	2	22	1	1	4
88	4th	2	6	0	12	0	1	3rd	4	5	0	20	0	1	2
89	3rd	3	5	1	16	1	0	4th	2	4	1	9	1	0	3
90	4th	2	4	1	9	1	1	3rd	6	2	0	12	1	0	4
91	2nd	5	3	1	16	1	1	1st	6	4	0	24	1	0	4
92	1st	2	5	1	11	1	1	2nd	4	2	3	11	1	1	4
93	2nd	6	5	3	33	0	1	1st	2	3	1	7	0	1	2
94	1st	5	6	1	31	1	0	2nd	2	6	0	12	1	0	4
95	4th	4	6	3	27	0	1	3rd	3	2	0	6	0	1	0
96	3rd	3	6	2	20	1	1	4th	5	3	1	16	1	1	3
97	3rd	2	5	0	10	0	1	4th	3	2	2	8	1	1	3
98	1st	6	5	5	35	1	0	2nd	3	5	2	17	0	1	1

Appendix V

Blocking Variables

Sex, school, and teacher variables were used as blocking variables and reduced the error mean square.

Sex - male or female, obviously.

School - two schools.

Teacher - four teachers, two in each school.

The following analysis of variance table gives pertinent information about these variables.

ANOVA

Source	df	MS	F	p <
School	1	.7488	1.8603	.1764
Teacher within school	2	.1806	.4488	.6400
Classroom (pooling of school and teacher/school)	3	.3700	.9193	.4354
Sex	1	.0033	.0081	.9284
Error Term	82	.4025		

There was no significant difference using variable #5 between groups of subjects based on sex, school or classroom.

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